Is Genuine Network Nonlocality exclusive to Pure States

Anantha Krishnan[1] , Anil Shaji[1] , Debashis Saha[1]
[1]Indian Institute of Science Education and Research, Thiruvananthapuram

Genuine Network Nonlocality

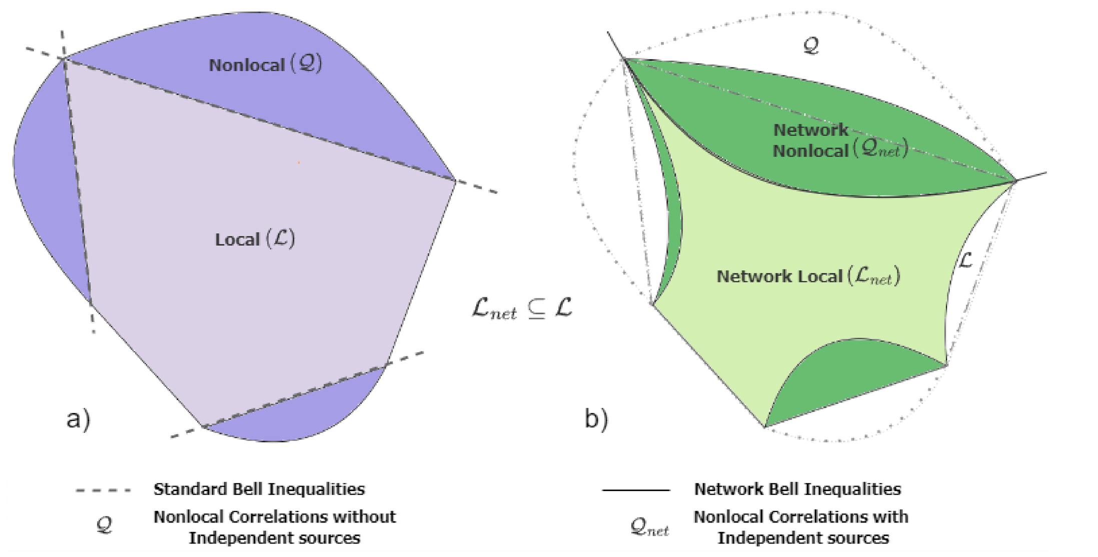

Fig. 1: a) Set of nonlocal correlations without independent sources b) Set of network nonlocal correlations with independent sources

Nonlocal correlations lie at the core of quantum foundations, and distributions displaying them necessitate novel properties and are of significant resource in information processing applications. Unlike quantum systems with a convex local boundary, networks enable novel correlations without inputs which is of particular interest as it deviates from the standard bell inequality violation criteria. Thus this study opens up new layers of complexity, revealing how quantum correlations can be optimized and exploited in practical applications. In this study, we focus on the triangle quantum network with X Mixed states to uncover novel properties of genuine network nonlocality expression and how its noise robustness is different from standard bell nonlocality.

$$P_Q(a,b,c) \neq \int d\alpha \int d\beta \int d\gamma \; P_A(a|\beta,\gamma)\, P_B(b|\gamma,\alpha)\, P_C(c|\alpha,\beta)$$

Network locality constraint (with independent sources)

Methodology: Encoding the Triangle Network in a Neural Network

With the triangle network, three sources α, β, γ send information through a quantum channel to the parties Alice, Bob & Charlie. The flow of information is constrained such that the sources are independent from each other, making the local set non-convex, in turn making the problem hard to formalize. Here with entangled sources and entangled measurements, the joint probability distributions exhibits genuine network nonlocal correlations. These correlations being irreproducible by a classical structure, a sufficiently expressive machine learning model respecting the causal probability relations of the network can inner approximate the local realistic joint probability distributions, thus characterizing irreproducible correlations as nonlocal.

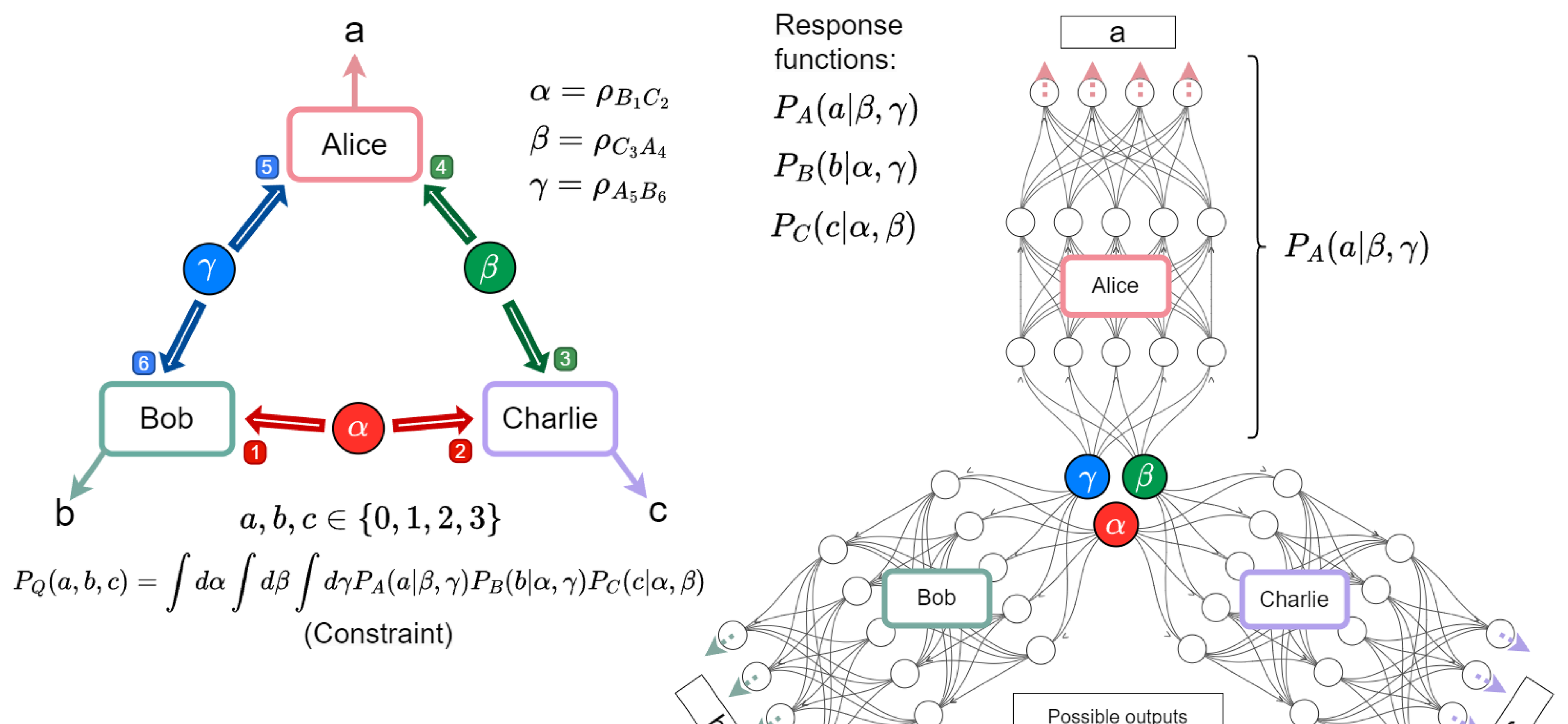

Fig. 2: Triangle network configuration and its neural network counterpart for single rank quantum state local distributions

The joint probability distributions of Mixed states are k sums of distributions, where k is the rank of the full density matrix. We find that adapting the model with this rank parameter lets us accurately probe the local landscape and study the noise robustness of GNN in the triangle scenario.

$$p(abc) = \sum_{k=1}^{n} p_k\, Tr(P_{AA}\rho^k_{AA})\, Tr(P_{BB}\rho^k_{BB})\, Tr(P_{CC}\rho^k_{CC}) \quad \text{where } p_k = \frac{1}{k}$$

For each individual model we adjust the weights of the neural network after evaluating the cost function on a batch size (Kullback divergence: which is a measure of discrepancy between the two distributions).

$$L(P_m) = \sum_{a,b,c} P_t(a,b,c)\; log\Big(\frac{P_t(a,b,c)}{P_m(a,b,c)}\Big)$$

$$P(a,b,c) = \frac{1}{k}\sum_{i=1}^{k} P_k(a,b,c)$$

This works parallelly to train the set of k models to learn the P(a,b,c) target distribution.

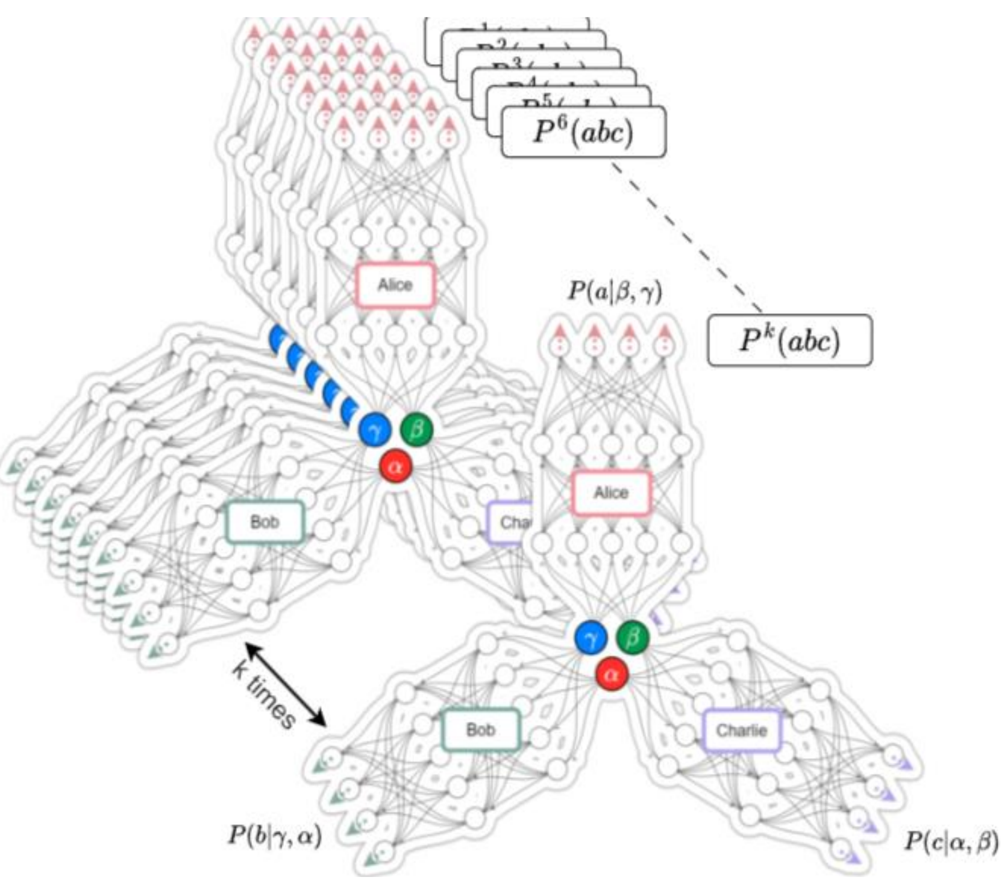

Fig. 3: The adapted neural network model for mixed states of rank k

X Mixed States and rank of the Global Quantum State

We generate the distribution using Bell Diagonal states

$$|\psi_\gamma\rangle_{A_\gamma B_\gamma} = |\psi_\alpha\rangle_{B_\alpha C_\alpha} = |\psi_\beta\rangle_{C_\beta A_\beta} = \begin{pmatrix} p & 0 & 0 & q \\ 0 & r & s & 0 \\ 0 & s & r & 0 \\ q & 0 & 0 & p \end{pmatrix}$$

with joint entangled measurements

$$u|00\rangle + (\sqrt{1-u^2})|11\rangle, (\sqrt{1-u^2})|00\rangle - u|11\rangle,$$
$$w|01\rangle + (\sqrt{1-w^2})|10\rangle, (\sqrt{1-w^2})|01\rangle - w|10\rangle$$

$$P_Q(a,b,c) = \langle M_a|\langle M_b|\langle M_c|\psi_g|M_a^t\rangle|M_b^t\rangle|M_c^t\rangle$$

We then train the neural network model using this target distribution (local distribution by construction) and measure the discrepancy using a distance measure. And if the measure is above the error threshold the distributions is nonlocal.

$$d(p_t, p_m) = \sqrt{\sum_{a,b,c} [p_t(abc) - p_m(abc)]^2}$$

Our results show that within the distribution those with states of rank greater than one couldn't exhibit the GNN correlations, and the pure states which can, exhibit the maximum distance discrepancy with the measurement parameters $(w^2, u^2) = (0.500, 0.875)$ and $(0.875, 0.500)$

This result is not observed with standard bell nonlocality and adds to the novelty of these correlations. This thus raises the question "Whether GNN is exclusive to pure states?"

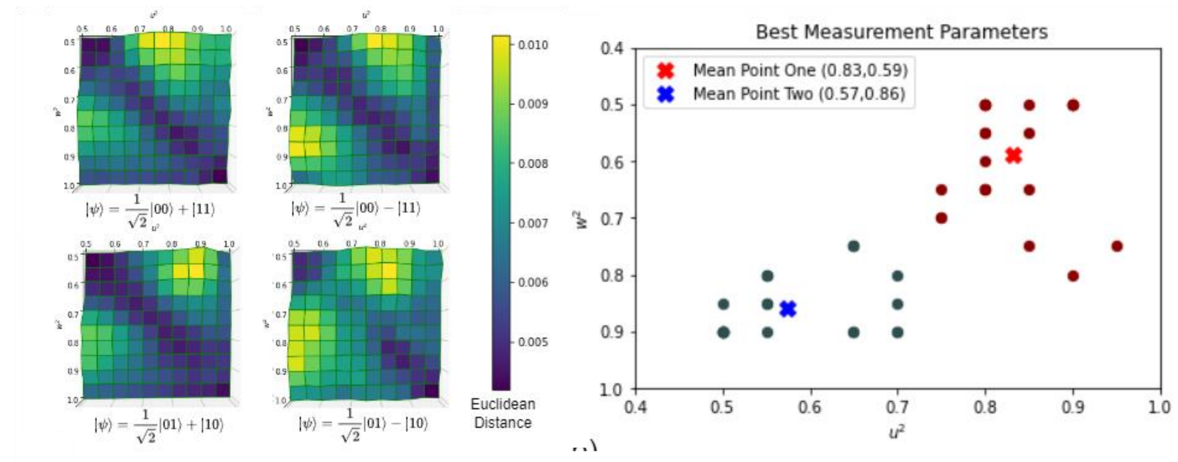

Fig. 4: a) Distance measure of the 4 Bell states (10^{-2} units) b) The best Measurement settings (w^2,u^2) = (0.500,0.875) & (0.875,0.500)

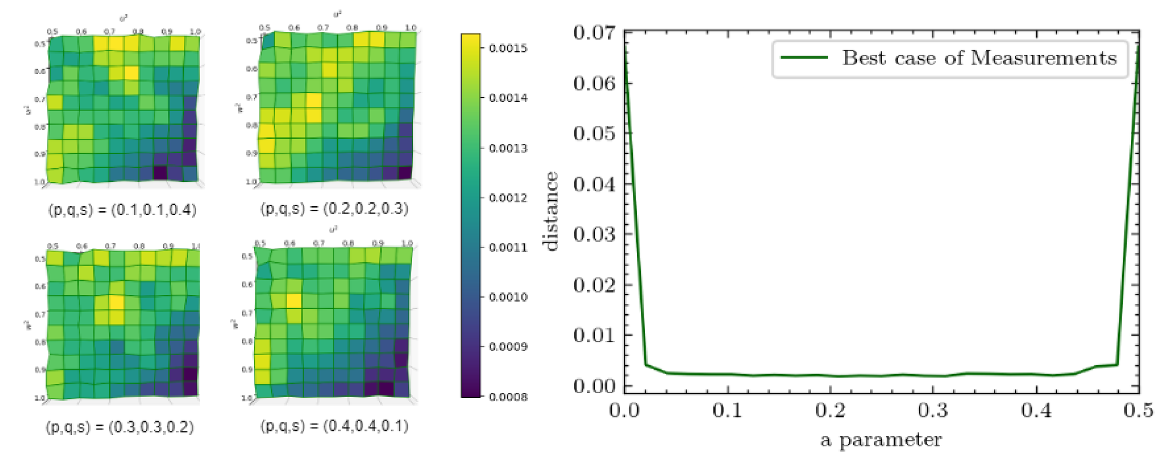

Fig. 5: a) Distance measure of X Mixed States (10^{-3} units) b) There are no occurrences of GNN except with Pure states.

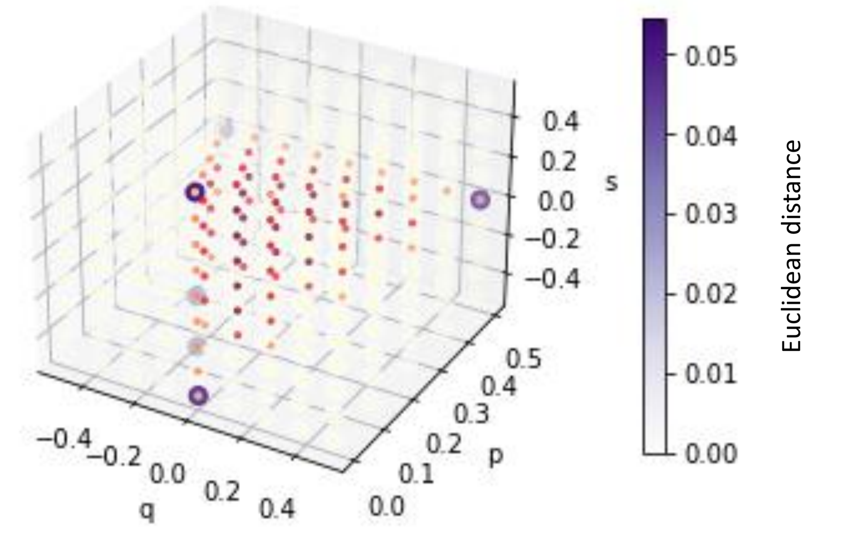

Fig. 6: Here the valid quantum states form the coloured tetrahedron with the four corners being Pure states. As proposed we can see that only the corners express GNN correlations

Noise Robustness

We explore noise robustness by adding Werner noise which takes the distribution towards the maximally mixed local state (I/4) using the visibility parameter v.

$$\rho(v) = v|\psi_-\rangle\langle\psi^-| + (1-v)I/4$$

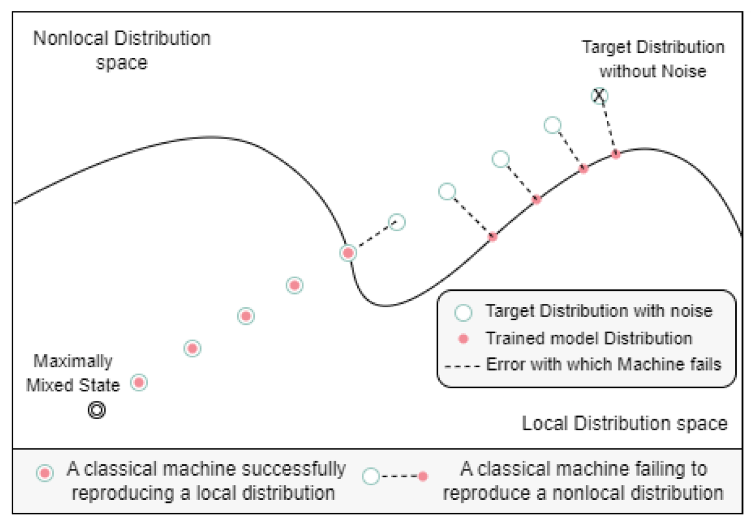

Fig. 7: Adding Werner noise moves any nonlocal distribution to the local side

We find that GNN disappears completely with the slightest addition of noise. This discrete loss of nonlocality is in contrast with the CHSH noise robust proof of standard bell nonlocality.

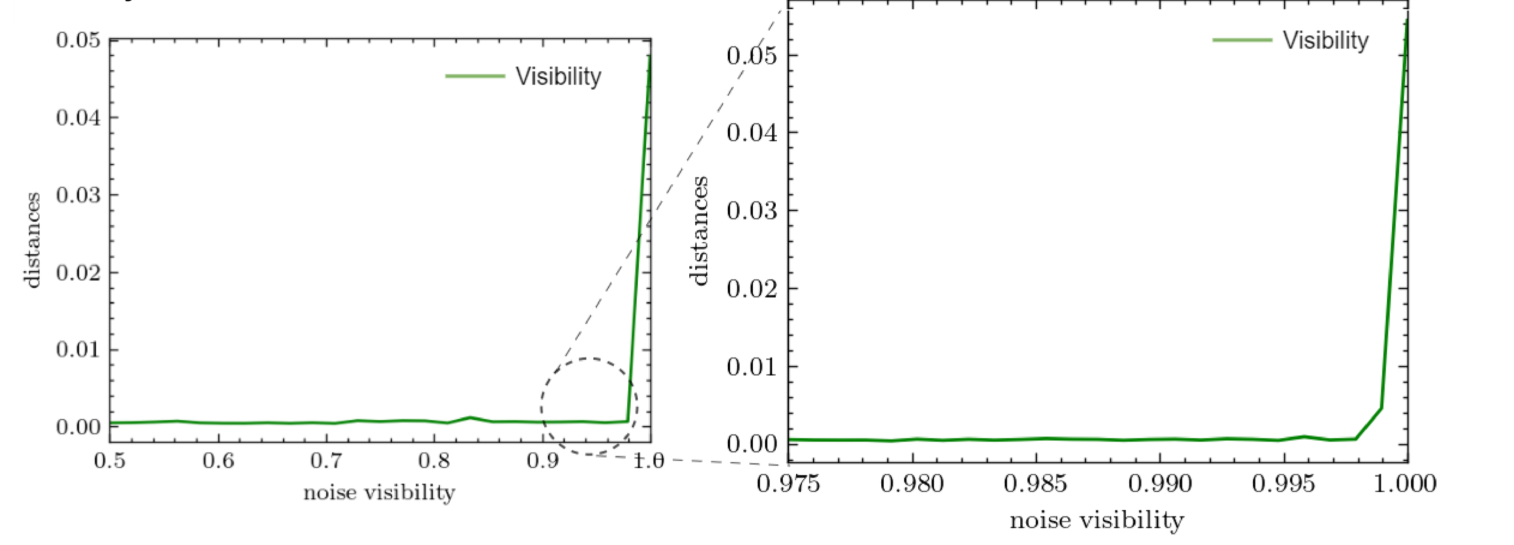

Fig. 8: Studying noise robustness taking all states equal with added Werner noise

We confirm that nonlocality cant be achieved up to 0.999, this limit is beyond any existing noise robustness studies where a local realistic model has been found.

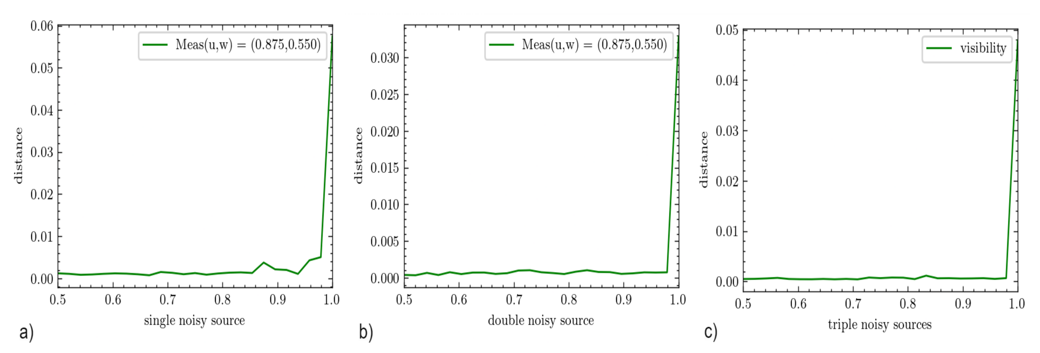

Fig. 9: Adding Werner noise to a) single sources b) double sources c) triple sources, GNN is only exhibited when the states are all noise free

Our results also show that each source has to be noise free to facilitate these correlations, meaning we need the full quantum state to be a pure state to exhibit GNN.

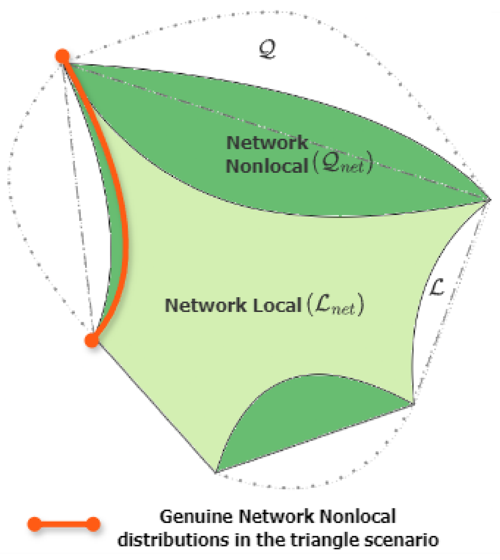

Fig. 10: Identifying GNN in the triangle scenario along the local boundary

With these results, we can identify the genuine network nonlocality correlations in the triangle scenario along the boundary(i.e. is closest to the local set), since with the slightest addition of noise, the distribution instantly falls into the local set. We can also identify quantum distributions without independent sources including those of Bell states towards the corners.

Conclusion

We find that a network local realistic model exists for all distributions with states of rank above one in the triangle scenario, this makes the correlations instantly disappear with the slightest noise. Thus GNN in the triangle scenario is a stronger set of correlation that is much more different than standard bell nonlocality in that it lies only in the boundary of the network local correlations. We have developed a rank based neural network model that can accurately characterize genuine network nonlocality to achieve these results.

References

Marc-Olivier Renou et al. "Genuine Quantum Nonlocality in the Triangle Network". In: Phys.Rev. Lett. 123.14 (2019), p. 140401.doi:10.1103/PhysRevLett.123.140401
Kriváchy, T., Cai, Y., Cavalcanti, D. *et al.* A neural network oracle for quantum nonlocality problems in networks. *npj Quantum Inf* **6**, 70 (2020). DOI:10.1038/s41534-020-00305-x